Rewarding Learning

ADVANCED
General Certificate of Education
2024

Further Mathematics

Assessment Unit A2 2

assessing

Applied Mathematics

[AFM21]

WEDNESDAY 5 JUNE, AFTERNOON

MARK SCHEME

General Marking Instructions

GCE Advanced/Advanced Subsidiary Further Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters M, W and MW as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of following through their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from a candidate's inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

Section A Mechanics 1

AVAILABLE MARKS

1

Shape	Rectangle CDEF	Triangle PFC	Triangle PAB	Circle	
Mass	$12 \times 24m$ $= 288m$	$\frac{1}{2} \times 12 \times 18m$ $=108m$	$\frac{1}{2} \times 4 \times 6m$ $=12m$	$\pi \times 3^2$ $= 9\pi m$	M1 W1
Distance of CoM from DE	12	$24 + \frac{1}{3} \times 18$ $= 30$	$36 + \frac{1}{3} \times 6$ $= 38$	10	M1 W1

$\Rightarrow 288m \times 12 + 108m \times 30 - 12m \times 38 - 9\pi m \times 10 =$ M2

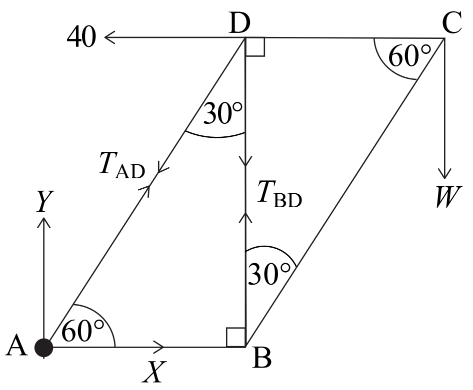
$(288m + 108m - 12m - 9\pi m)\,\bar{y}$ W2

$6240 - 90\pi = (384 - 9\pi)\,\bar{y}$

$\bar{y} = 16.7$ cm (3 sf) W1 9

AVAILABLE MARKS

2 **(i)**



Whole system

M(A) $W \times 2l = 40 \times \sqrt{3}l$ M1 W2

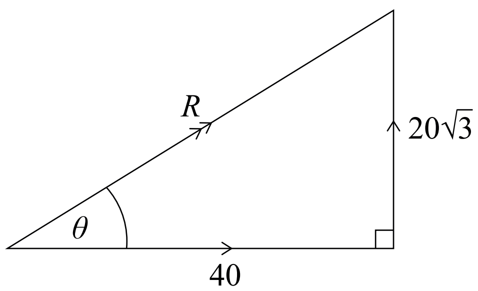
$W = 20\sqrt{3}$ W1

(ii) Whole system

(↑) $Y = W$

$Y = 20\sqrt{3}$ M1 W1

(→) $X = 40$ MW1



$R = \sqrt{40^2 + \left(20\sqrt{3}\right)^2}$ M1

$R = 52.9\,\text{N}$ (3 sf) W1

$\theta = \tan^{-1}\left(\dfrac{20\sqrt{3}}{40}\right)$ M1

$\theta = 40.9°$ (3 sf) W1

i.e. 52.9 N at 40.9° to the horizontal

(iii) A (↑) $T_{AD}\ \cos 30° + Y = 0$ M1

$T_{AD} = -40\,\text{N}$ W1

D (↓) $T_{AD}\ \cos 30° + T_{BD} = 0$ M1

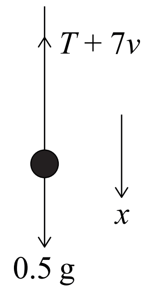
$T_{BD} = 20\sqrt{3}\ \text{N}$ W1

(iv) BD is in tension. MW1 16

			AVAILABLE MARKS
3	**(i)** $T = mg$	M1	
	$\frac{5mge}{l} = mg$		
	$e = \frac{l}{5}$ metres	W1	
	(ii) $mg - T = m\frac{d^2x}{dt^2}$	M1 W1	
	$mg - \frac{5mg(e+x)}{l} = m\frac{d^2x}{dt^2}$	M1	
	$mg - \frac{5mge}{l} - \frac{5mgx}{l} = m\frac{d^2x}{dt^2}$	W2	
	$\frac{d^2x}{dt^2} = -\frac{5g}{l}x$	W1	
	(iii) Maximum value of x is $\frac{l}{5}$	MW1	
	(iv) $t = 0$, $v = 0$, $x = 0.2$		
	$\Rightarrow$ Amplitude is 0.2 m	MW1	
	$\omega^2 = \frac{5g}{1.4}$	M1	
	$\omega = \sqrt{35}$	W1	
	$x = r\cos\omega t$		
	$\Rightarrow x = 0.2\cos\left(\sqrt{35}\,t\right)$	M1	
	$x = 0.15 \Rightarrow 0.15 = 0.2\cos\left(\sqrt{35}\,t\right)$	W1	
	$t = 0.122$ (3 sf)	W1	15

AVAILABLE MARKS

4 (i)



$0.5\mathrm{g} - T - 7v = 0.5a$ M1 W1

$5 - \frac{16x}{0.8} - 7\frac{\mathrm{d}x}{\mathrm{d}t} = 0.5\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ MW1

$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 14\frac{\mathrm{d}x}{\mathrm{d}t} + 40x = 10$ W1

(ii) AQE: $m^2 + 14m + 40 = 0$ M1 W1

$b^2 - 4ac = 196 - 160$ M1

$\Rightarrow b^2 - 4ac > 0$

Hence the motion is over-damped. MW1

(iii) $m^2 + 14m + 40 = 0$

$(m + 4)(m + 10) = 0$

$m = -4, -10$ MW1

CF: $x = Ae^{-4t} + Be^{-10t}$ MW1

PI: $x = \mathrm{C}$

$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 0$ M1

$\Rightarrow 0 + 0 + 40C = 10$

$C = \frac{1}{4}$ W1

$\Rightarrow x = Ae^{-4t} + Be^{-10t} + \frac{1}{4}$

$t = 0, x = 0 \Rightarrow 0 = A + B + \frac{1}{4}$ ① M1

$t = 0, v = 0 \quad v = -4Ae^{-4t} - 10Be^{-10t}$

$\Rightarrow 0 = -4A - 10B$ ② M1

$0 = 4A + 4B + 1$ 4 × ①

$0 = -6B + 1$

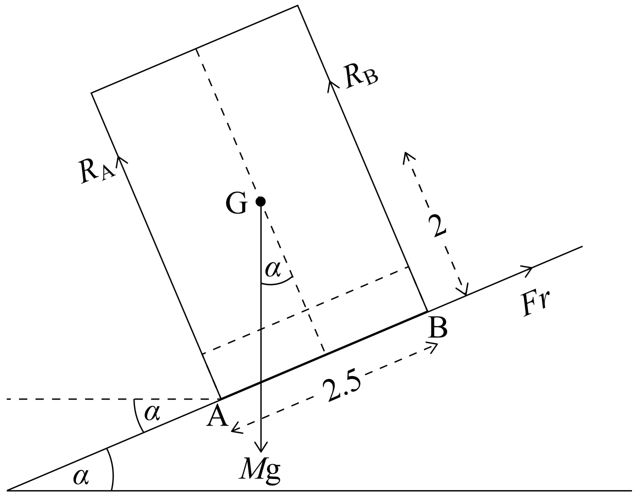
$B = \frac{1}{6}$ W1

$A = -\frac{5}{12}$ W1

$\Rightarrow x = -\frac{5}{12}e^{-4t} + \frac{1}{6}e^{-10t} + \frac{1}{4}$

$t = 1 \quad x = 0.242$ (3 sf) W1 17

5 (i)

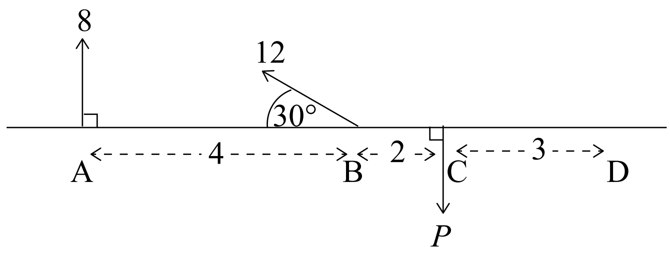


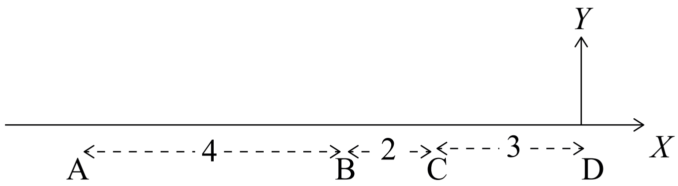
			AVAILABLE MARKS
About to topple inwards $\Rightarrow R_B = 0$		MW1	
M(G)	$R_A \times 1.25 = Fr \times 2$	M1 W1	
	$Fr = 0.625R_A$	W1	
(←)	$R_A \sin\alpha - Fr\cos\alpha = \dfrac{Mv^2}{r}$	M3 W1	
(↑)	$R_A\cos\alpha + Fr\sin\alpha = Mg$	M1 W1	
$\Rightarrow \dfrac{R_A\sin\alpha - 0.625R_A\cos\alpha}{R_A\cos\alpha + 0.625R_A\sin\alpha} = \dfrac{Mv^2}{Mrg}$		M3	
$\Rightarrow v^2 = rg\left(\dfrac{\tan\alpha - 0.625}{1 + 0.625\tan\alpha}\right)$		W1	
(ii) $\alpha = 25 \Rightarrow v^2 = rg\left(\dfrac{\tan 25° - 0.625}{1 + 0.625\tan 25°}\right)$			
$\tan 25° - 0.625 < 0$		MW1	
$\Rightarrow v$ not feasible			
$\Rightarrow$ It is not possible to be on the point of toppling inwards.		MW1	
$\alpha = 25, \mu = 0.4 \Rightarrow u^2 = rg\left(\dfrac{\tan 25° - 0.4}{1 + 0.4\tan 25°}\right)$			
$\tan 25° - 0.4 > 0$		MW1	
$\Rightarrow u$ is feasible			
$\Rightarrow$ It is possible to slide downwards.		MW1	18
		Section A	**75**

Section B Mechanics 2

AVAILABLE MARKS

1 **(i)**



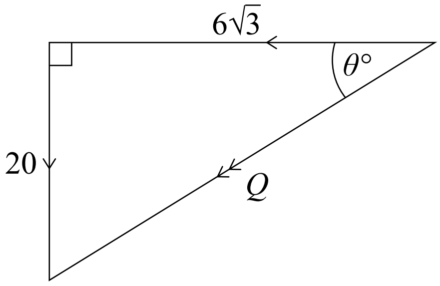


M(C) $\quad 8 \times 6 + 12 \cos 60° \times 2 = -3Y$ — M1 W1

$Y = -20$ — W1

($\rightarrow$) $\quad -12 \cos 30° = X$ — M1

$X = -6\sqrt{3}$ — W1



$Q = \sqrt{(6\sqrt{3})^2 + 20^2}$ — M1

$Q = 22.5$ (3 sf) — W1

$\theta = \tan^{-1}\left(\frac{20}{6\sqrt{3}}\right)$ — M1

$= 62.5°$ (3 sf) below line DA — W1

(ii) ($\uparrow$) $\quad 8 + 12 \cos 60° - P = -20$ — M1

$P = 34$ — W1 — 11

			AVAILABLE MARKS
2	**(i)** $t = 2$ $\mathbf{v} = \mathbf{u} + \mathbf{a}t$		
	$\mathbf{v} = (7\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) + 2(4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$	M2	
	$= 15\mathbf{i} + 8\mathbf{j} + 6\mathbf{k}$	W1	
	$2 < t \leqslant 4$ $\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t$	M1	
	$\mathbf{v} = 7t\mathbf{i} - 8t^{-2}\mathbf{j} + t^2\,\mathbf{k} + \mathbf{c}$	W1	
	$t = 2$		
	$\Rightarrow 15\mathbf{i} + 8\mathbf{j} + 6\mathbf{k} = 14\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \mathbf{c}$	M1 W1	
	$\mathbf{c} = \mathbf{i} + 10\mathbf{j} + 2\mathbf{k}$		
	$\mathbf{v} = (7t + 1)\mathbf{i} + \left(10 - \frac{8}{t^2}\right)\mathbf{j} + (t^2 + 2)\mathbf{k}$	W1	
	(ii) $\mathbf{s} = \int \mathbf{v}\,\mathrm{d}t$		
	$= \left(\frac{7}{2}t^2 + t\right)\mathbf{i} + \left(10t + \frac{8}{t}\right)\mathbf{j} + \left(\frac{1}{3}t^3 + 2t\right)\mathbf{k} + \mathbf{c}$	M1 W1	
	$t = 2$ $\mathbf{s} = 22\mathbf{i} + 10\mathbf{j} + 16\mathbf{k}$		
	$\Rightarrow 22\mathbf{i} + 10\mathbf{j} + 16\mathbf{k} = 16\mathbf{i} + 24\mathbf{j} + \frac{20}{3}\mathbf{k} + \mathbf{c}$	M1	
	$\mathbf{c} = 6\mathbf{i} - 14\mathbf{j} + \frac{28}{3}\mathbf{k}$	W1	
	$\mathbf{s} = \left(\frac{7}{2}t^2 + t + 6\right)\mathbf{i} + \left(10t + \frac{8}{t} - 14\right)\mathbf{j} + \left(\frac{1}{3}t^3 + 2t + \frac{28}{3}\right)\mathbf{k}$		
	$t = 4 \Rightarrow$ $\mathbf{s} = 66\mathbf{i} + 28\mathbf{j} + \frac{116}{3}\mathbf{k}$	MW1	13

				AVAILABLE MARKS
3	**(i)**	$m_A = m \qquad m_B = m$ $u_A = u \qquad u_B = 0$		
		$mu + 0 = mv_A + mv_B$		
		$u = v_A + v_B$	M1 W1	
		$-\frac{1}{2}u = v_A - v_B$	M1 W1	
		$v_A = \frac{1}{4}u \qquad v_B = \frac{3}{4}u$	M1 W2	
		$m_B = m \qquad m_C = m$ $u_B = \frac{3}{4}u \qquad u_C = 0$		
		$\frac{3}{4}mu + 0 = mv_B + mv_C$	MW1	
		$\frac{3}{4}u = v_B + v_C$		
		$-\frac{3}{8}u = v_B - v_C$	MW1	
		$v_B = \frac{3}{16}u \qquad v_C = \frac{9}{16}u$	MW2	
		After 2nd collision $v_A = \frac{1}{4}u \qquad v_B = \frac{3}{16}u \qquad v_C = \frac{9}{16}u$		
	(ii)	A and B are moving in the same direction and $v_A > v_B$ Hence A and B will collide for a second time.	MW1	
	(iii)	$m_A = m \qquad m_B = m$ $u_A = \frac{1}{4}u \qquad u_B = \frac{3}{16}u$		
		$\frac{1}{4}mu + \frac{3}{16}mu = mv_A + mv_B$	MW1	
		$\frac{7}{16}u = v_A + v_B$		
		$-\frac{1}{32}u = v_A - v_B$	MW1	
		$v_A = \frac{13}{64}u \qquad v_B = \frac{15}{64}u \qquad v_C = \frac{9}{16}u$	M1 W1	
		A, B and C are moving in the same direction with $v_A < v_B$ and $v_B < v_C$	MW1	
		Therefore no further collisions will happen.	MW1	18

				AVAILABLE MARKS
4	(i)	$40t - 8(v + 1) = 4a$	M1 W1	
		$\Rightarrow \dfrac{dv}{dt} = 10t - 2(v + 1)$		
		$\Rightarrow \dfrac{dv}{dt} + 2v = 10t - 2$	MW1	
	(ii)	$\text{IF} = e^{\int 2dt} = e^{2t}$	M1 W1	
		$\Rightarrow e^{2t}\dfrac{dv}{dt} + 2e^{2t}v = (10t - 2)e^{2t}$	MW1	
		$ve^{2t} = \int (10t - 2)\, e^{2t}\, dt$	M1 W1	
		$U = 10t - 2 \qquad \dfrac{dV}{dt} = e^{2t}$	M1	
		$\dfrac{dU}{dt} = 10 \qquad V = \dfrac{1}{2}e^{2t}$	W1	
		$\Rightarrow ve^{2t} = (5t - 1)\, e^{2t} - \int 5e^{2t}\, dt$	M1 W1	
		$ve^{2t} = (5t - 1)\, e^{2t} - \dfrac{5}{2}e^{2t} + c$	MW1	
		$t = 0,\ v = 0 \qquad 0 = -1 - \dfrac{5}{2} + c$	M1	
		$c = \dfrac{7}{2}$		
		$v = (5t - 1) - \dfrac{5}{2} + \dfrac{7}{2}e^{-2t}$	W1	
		$v = 5t - \dfrac{7}{2} + \dfrac{7}{2}e^{-2t}$		
		$t = 6 \quad v = 26.5\ \text{m s}^{-1} \quad (3\ \text{sf})$	MW1	16

5 (i)

Shape	Cylinder	Removed hemisphere	Added hemisphere	
Mass	$\pi \times (2R)^2 \times 6R \times \rho$ $= 24\pi\rho R^3$	$\frac{2}{3}\pi \times R^3 \times \rho$ $= \frac{2}{3}\pi\rho R^3$	$\frac{2}{3}\pi \times R^3 \times 3\rho$ $= 2\pi\rho R^3$	M2 W2
Distance of CoM from base	$3R$	$6R - \frac{3}{8}R$ $= \frac{45}{8}R$	$6R - \frac{3}{8}R$ $= \frac{45}{8}R$	M1 W1

$$24\pi\rho R^3 \times 3R - \frac{2}{3}\pi\rho R^3 \times \frac{45}{8}R + 2\pi\rho R^3 \times \frac{45}{8}R$$
$$= \left(24 - \frac{2}{3} + 2\right)\pi\rho R^3\bar{y}$$ M1 W1 MW1

$$\frac{159}{2}\pi\rho R^4 = \frac{76}{3}\pi\rho R^3\bar{y}$$

$$\bar{y} = \frac{477}{152}R \text{ metres } (= 3.14R)$$ W1

(ii) About to slide

$Fr = \mu N$

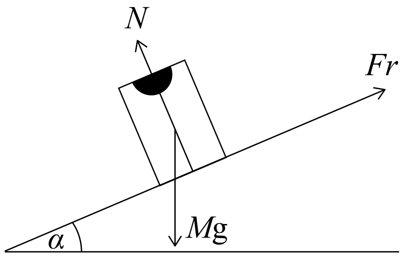
(↗) $Fr = Mg \sin\alpha$

(↖) $N = Mg \cos\alpha$ MW1

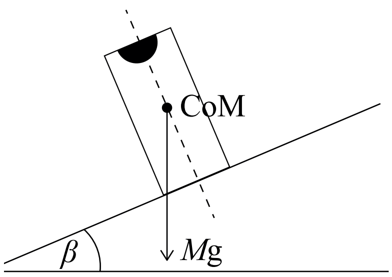
$\Rightarrow 0.5\, Mg \cos\alpha = Mg \sin\alpha$ M1

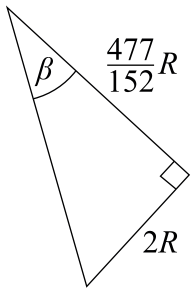
$\tan\alpha = 0.5$

$\alpha = 26.6°$ W1



About to topple





$$\tan\beta = \frac{2R}{\left(\frac{477}{152}R\right)}$$ M1 MW1

$$\beta = 32.5°$$ W1

Since the angle of the slope is only 20° and therefore less than α or β the ornament neither topples nor slides. MW1

AVAILABLE MARKS
17
Section B **75**

SECTION C Statistics

AVAILABLE MARKS

1 **(i)** Let $X = 2F - 3S$

$E(X) = 2E(F) - 3E(S)$

$E(X) = 2 \times 14 - 3 \times 10$ M1

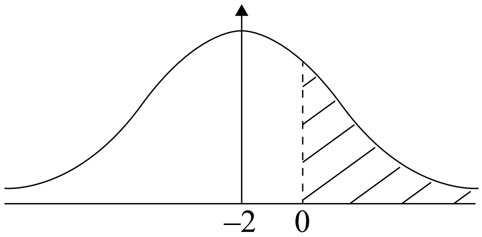
$E(X) = -2$ W1

$\text{Var}(X) = 2^2 \times \text{Var}(F) + 3^2 \times \text{Var}(S)$

$\text{Var}(X) = 4 \times 2 + 9 \times 1$ M1 MW1

$\text{Var}(X) = 17$ W1

$X \sim N(-2, 17)$ MW1



$P\left(S < \frac{2}{3}F\right) = P\left(\frac{2}{3}F - S > 0\right)$

$= P(2F - 3S > 0)$

$= P(X > 0)$ M1

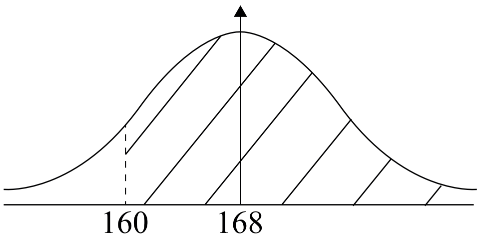
$= 0.314$ M1 W1

(ii) $F \sim N(14, 2)$

Let $T = F_1 + F_2 + F_3 + \ldots + F_{12}$

$E(T) = 12E(F)$

$E(T) = 12 \times 14 = 168$ MW1



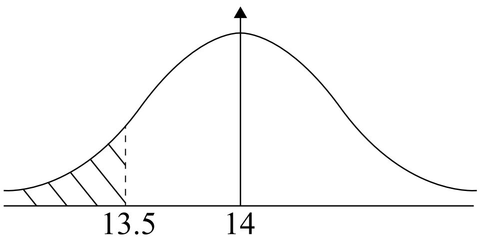
$\text{Var}(T) = 12 \times 2 = 24$ MW1

$T \sim N(168, 24)$

$P(T > 160) = 0.949$ M1 W1

(iii) $\bar{F} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$\bar{F} \sim N\left(14, \frac{2}{12}\right)$ M1 W1



$P(\bar{F} < 13.5) = 0.110$ M1 W1 17

2 **(i)** H_0 : student gender and likelihood to study Chemistry are independent MW1
H_1 : student gender and likelihood to study Chemistry are not independent MW1

AVAILABLE MARKS

Observed frequencies

	Chose Chem	Did not choose Chem	Total
Male	54	26	80
Female	64	56	120
Total	118	82	200

M1 W1

If H_0 is true then the expected number of male students who choose Chemistry is

$\frac{80 \times 118}{200} = 47.2$ M1 W1

Expected frequencies

	Chose Chem	Did not choose Chem	Total
Male	47.2	32.8	80
Female	70.8	49.2	120
Total	118	82	200

M1 W1

Test statistic: $\chi^2 = \sum \frac{(|O - E| - 0.5)^2}{E}$ (applying Yates' correction)

O	E	$\frac{(\lvert O - E\rvert - 0.5)^2}{E}$
54	47.2	0.8409
26	32.8	1.2101
64	70.8	0.5606
56	49.2	0.8067

M1 MW1
W1

$\chi^2 = 3.4183$ W1

2×2 contingency table i.e. $h = 2$ and $k = 2$
$v = 1 \times 1 = 1$ MW1
Critical value: $\chi^2_{10\%}(1) = 2.705$ MW1

$\chi^2 > \chi^2_{10\%}(1) \therefore$ reject H_0 MW1
i.e. there is sufficient evidence, at the 10% level, to indicate that student gender and likelihood to choose Chemistry are not independent. MW1

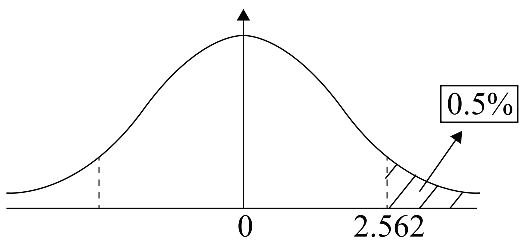
(ii) From observed frequencies.
Percentage of males who chose Chemistry = $\frac{54}{80} = 67.5\%$
Percentage of females who chose Chemistry = $\frac{64}{120} = 53.3\%$
Given the lack of independence in part **(i)**, Chemistry teacher may wish to focus on female students. MW1
Smaller proportion of female students chose Chemistry. MW1

18

3 **(i)** Before refinements

		AVAILABLE MARKS
$\overline{x} = \dfrac{\sum x}{n} = \dfrac{1282.4}{8} = 160.3$	MW1	
$S_x^2 = \dfrac{n}{n-1}\left(\dfrac{\sum x^2}{n} - \overline{x}^2\right) = \dfrac{8}{7}\left(\dfrac{205586.08}{8} - 160.3^2\right) = 2.48$	M1 W1	
After refinements $S_y^2 = \dfrac{\sum(y-\overline{y})^2}{n-1} = \dfrac{6.336}{9} = 0.704$	M1 W1	
Pooled estimator $S_{\mathrm{P}}^2 = \dfrac{(7 \times 2.48)+(9 \times 0.704)}{16} = 1.481$	M1 W1	
(ii) $\mathrm{H}_0 : \mu_x = \mu_y$	MW1	
$\mathrm{H}_1 : \mu_x \neq \mu_y$	MW1	
Two-sample *t*-test (two-tailed)	M1	
$v = 16 \therefore T \sim t(16)$	MW1	
Test statistic: $T = \dfrac{(\overline{X}-\overline{Y})-(\mu_x-\mu_y)}{\sqrt{S_{\mathrm{P}}^2\left(\dfrac{1}{n_x}+\dfrac{1}{n_y}\right)}}$	M1	
If H_0 is true then $\mu_x - \mu_y = 0$, and the test value is given by		
$t_{\text{test}} = \dfrac{\overline{x}-\overline{y}}{\sqrt{S_{\mathrm{P}}^2\left(\dfrac{1}{n_x}+\dfrac{1}{n_y}\right)}} = \dfrac{160.3 - 160.08}{\sqrt{1.481\left(\frac{1}{8}+\frac{1}{10}\right)}} = 0.381$	MW1 W1	
Critical value at 10% (5% in each tail) with $v = 16$ is $t_c = 1.746$	MW1	
$\lvert t_{\text{test}}\rvert < t_c \therefore$ do not reject H_0	MW1	
i.e. there is insufficient evidence, at the 10% level, to claim that the refinements have affected the average volume dispensed.	MW1	
(iii) Samples taken from Normally distributed populations.	MW1	
(iv) The two samples are taken from populations with equal variances.	MW1	
Population variance after refinement likely to be smaller than that before.	MW1	20

			AVAILABLE MARKS
4	**(i)** $S^2 = \dfrac{78.75}{n-1}$	M1 W1	
	$0.98 = 2 \times 1.96 \times \dfrac{S}{\sqrt{n}}$	M2 MW1	
	$0.0625 = \dfrac{S^2}{n}$		
	$0.0625 = \dfrac{78.75}{n(n-1)}$	M1	
	$0.0625n^2 - 0.0625n - 78.75 = 0$	W1	
	$n^2 - n - 1260 = 0$		
	$n = 36$		
	$S^2 = 2.25$	MW1	
	(ii) $\bar{x} = 4.61 + 0.49 = 5.1$	MW1	
	90% CI $= 5.1 \pm 1.645 \times \dfrac{1.5}{\sqrt{36}}$	M1 MW1	
	$= (4.69, 5.51)$ (3 sf)	W1	
	(iii) Answer would be the same.	MW1	
	Large sample ($n > 30$) $\therefore$ CLT applies.	MW1	
	(iv) 90% of 80 = 72	MW1	
	(v) $0.98 = 2 \times z \times \dfrac{1.6}{\sqrt{70}}$	MW1	
	$z = 2.562$	W1	
	From tables $P(Z < 2.562) = 0.9948$	M1 W1	
	$(1 - 0.9948) \times 2 = 0.0104 = 1.04\%$		
	i.e. 99% CI	MW1	20

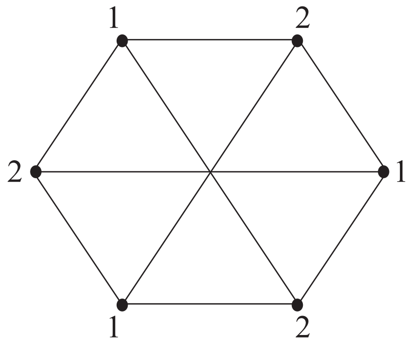


	Section C	**75**

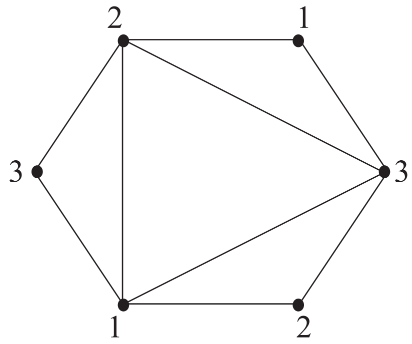
SECTION D Discrete and Decision Mathematics

AVAILABLE MARKS

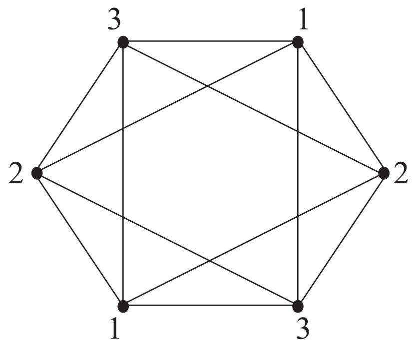
1 (a) (i)



A



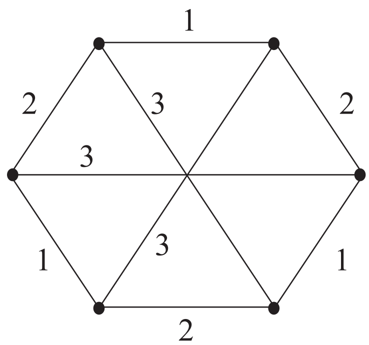
B



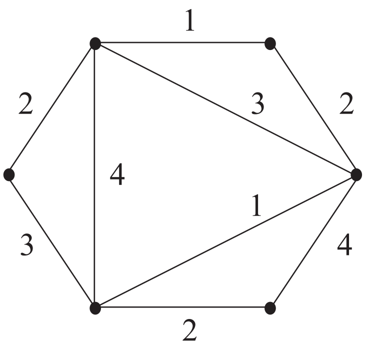
C

M1 W3

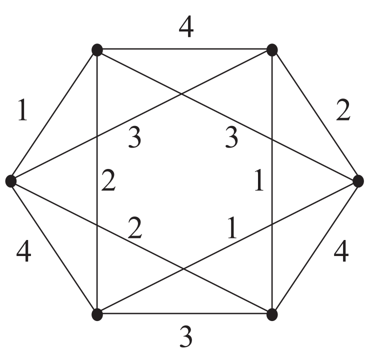
(ii)



A



B



C

M1 W3

AVAILABLE MARKS

(b) (i) A cutset is a set of edges whose removal partitions the graph into two disjoint subgraphs L and R. M1

Each edge in the cutset joins a vertex of L to a vertex of R. W1

(ii) { [be], [bf], [cf], [df], [dg] } M1 W1

(iii) Maximum flow from vertex a to vertex h = sum of capacities of edges in the minimum cutset = 9 M1 W1 14

2 (i) s → t → v → w → u → q → p → r M1 W2

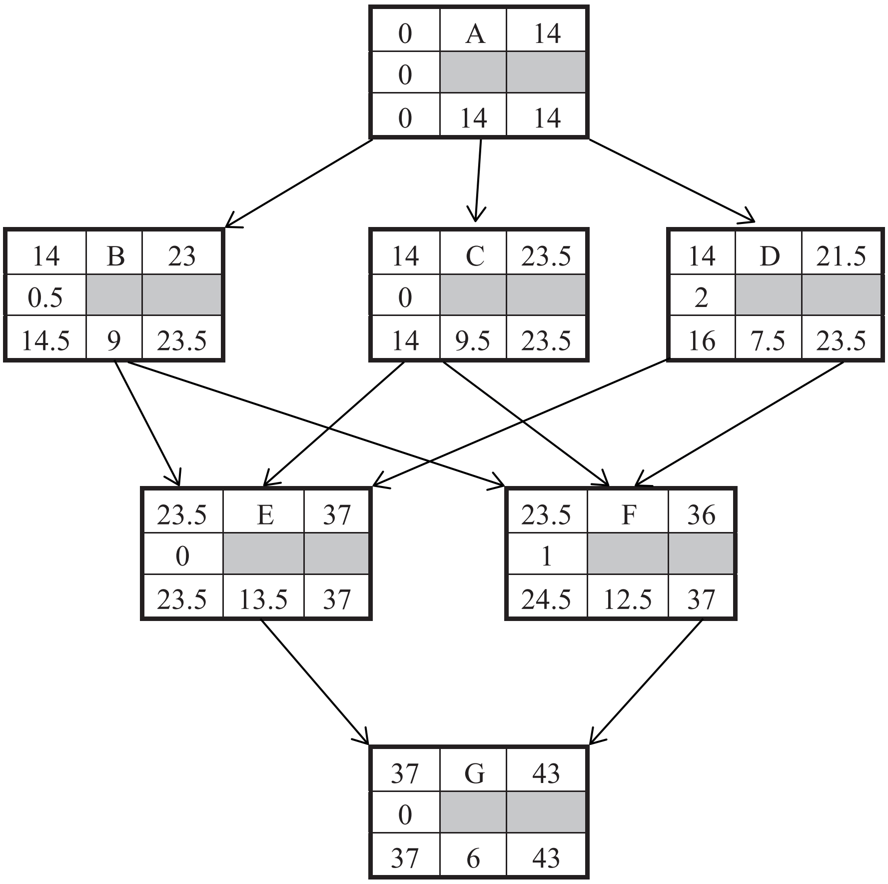
(ii) 17 MW1

(iii) v → t → s → p → r → u → q M1 W1

(iv) It does not guarantee traversing every vertex to complete a path. MW1 7

3 (i) 13.5, 12.5 weeks M1 W1

(ii)



EF times MW1
LS times MW2
LF times MW2
Slack times MW1

(iii) Critical path = A C E G MW1

(iv)

Activity	Predecessor	t-opt	t-norm	t-pess	t-exp
A	–	11	14	17	14
B	A	7	9	11	9
C	A	8	9	13	9.5
D	A	5	7	12	7.5
E	B, C, D	10	14	15	13.5
F	B, C, D	11	12	16	12.5
G	E, F	$6 - d$	6	$6 + d$	6

Expected time = 14 + 9.5 + 13.5 + 6 = 43 weeks MW1

Project variance = $(36 + 25 + 25 + 4d^2)/36$ M2 W1

$P(T < 46) = 0.95$

$z = 1.645$ MW1

$$z = \frac{(46 - 43)}{\left(\frac{\sqrt{86 + 4d^2}}{6}\right)} = 1.645$$ M1 W1

$$86 + 4d^2 = \left(\frac{3 \times 6}{1.645}\right)^2$$

$d = 2.90$ W1

pessimistic time = 6 + 2.90 = 8.90 weeks W1 18

4 **(i)** $(1 + x + x^2)^3 = (1 + x + x^2)(1 + x + x^2)^2$ M1

$= (1 + x + x^2)(1 + 2x + 3x^2 + 2x^3 + x^4)$ W1

$= 1 + 3x + 6x^2 + 7x^3 + 6x^4 + 3x^5 + x^6$ W1

3 items = 7 ways M1 W1

(ii) $(x + x^2 + x^3)^2$ M1 W2 8

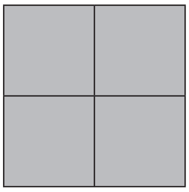
AVAILABLE MARKS

AVAILABLE MARKS

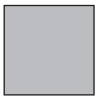
5 **(i)** 0 rooks 1 way
1 rook 4 ways
2 rooks 2 ways — M1 W1

So the rook polynomial is $1 + 4x + 2x^2$ — W1

(ii) Rook polynomial for



is $1 + 4x + 2x^2$ — M1

Rook polynomial for  is $1 + x$

MW1

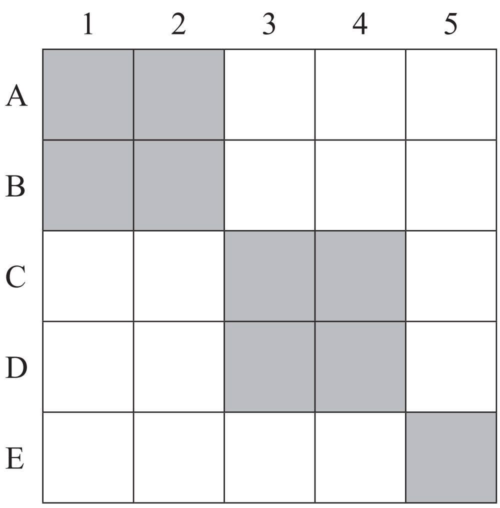
Three shapes are non-interfering. — M1

Combined rook polynomial is $(1 + x)(1 + 4x + 2x^2)(1 + 4x + 2x^2)$ — M1

$= (1 + x)(1 + 8x + 20x^2 + 16x^3 + 4x^4)$

$= 1 + 9x + 28x^2 + 36x^3 + 20x^4 + 4x^5$ — W2

(iii)



M1 W2

(iv) Rook polynomial for excluded positions
$= 1 + 9x + 28x^2 + 36x^3 + 20x^4 + 4x^5$ — MW1

Rook IE theorem

Number of positions
$= 5! - 9 \times 4! + 28 \times 3! - 36 \times 2! + 20 \times 1! - 4 \times 0!$ — M2 W2

16 ways — W1 — 18

			AVAILABLE MARKS
6	**(a)** $P_G = \frac{1}{12}(x_1^6 + 8x_3^2 + 3x_1^2x_2^2)$		
	Replace x_i by 3	M1	
	Number of possible colour patterns is $\frac{1}{12}(x_1^6 + 8x_3^2 + 3x_1^2x_2^2)$	MW1	
	$= \frac{1}{12}(3^6 + 8 \times 3^2 + 3 \times 3^2 \times 3^2)$		
	$\frac{1}{12}(729 + 72 + 243) = 87$	W1	
	(b) **(i)** period = 3	MW1	
	(ii) 2 non-identity elements × 4 sets	M1	
	= 8 elements	W1	
	(iii) period = 2	MW1	
	(iv) 1 non-identity element × 3 sets	M1	
	= 3 rotation elements	W1	
	(v) order = 12	MW1	10
		Section D	**75**
		Total	**150**